AP Calculus BC — Rates of Change

1) a) $A = \pi r^2$ $\quad C = 2\pi r$ $\quad r = \frac{C}{2\pi}$

$A = \pi\left(\frac{C}{2\pi}\right)^2$

$A(C) = \frac{C^2}{4\pi}$

b) $\frac{dA}{dC} = \frac{C}{2\pi}$

c) $\frac{dA}{dC} \rightarrow \frac{\text{square inches}}{\text{inches}}$

d) $\left.\frac{dA}{dC}\right|_{C=\pi} = \frac{1}{2}\ in^2/in$ $\quad \left.\frac{dA}{dC}\right|_{C=6\pi} = 3\ in^2/in$

2) $Q(t) = 200(30-t)^2$

$Q'(t) = 400(30-t)(-1)$

$Q'(10) = -400(20) = -8000\ gallons/minute$

At $t=10$ min. water is draining from the tank at a rate of 8000 gal/min

$\frac{Q(10)-Q(0)}{10-0} = \frac{(200)(400)-200(900)}{10} = -10000\ gal/min$

From $t=0$ to $t=10$ min. water has drained from the tank at an avg. rate of 10000 gal/min

3) $V = \frac{4}{3}\pi r^3$

a) $\frac{dV}{dr} = 4\pi r^2$

$\left.\frac{dV}{dr}\right|_{r=2} = 16\pi\ ft^3/ft$

b) $V(2.2) - V(2) = 11.092\ ft^3$

$\Delta V = 11.092\ ft^3$

4) a) $S''(10) \approx \frac{S'(12)-S'(8)}{12-8} = \frac{1.7-1.9}{4}\ cm/hr^2$

At $t=10$ hour, the rate of change of the depth of the snow is decreasing at a rate of $1/20\ cm/hr^2$.

b) Since $S(t)$ is differentiable, $S'(t)$ is continuous.

$S'(0) = 2.3 > 2$

$S'(14) = 1.6 < 2$

$\therefore$ By IVT, $S'(t) = 2$ in $(0, 14)$.

c) $S(8) = 45 \quad S'(8) = 1.9$

$L(t) = 45 + 1.9(t-8)$

$S(10) \approx L(10) = 45 + 1.9(2)$

$= 48.8$ cm

$S(t)$ is concave down, so $L(t) > S(t)$, $\therefore L(10) > S(10)$

d) $D(t) = 120 - 92e^{-t/40}$

$D'(t) = -92e^{-t/40}\left(-\frac{1}{40}\right)$

$D'(10) = \frac{23}{10}e^{-1/4}$ cm/hr

At $t = 10$ hrs. the depth of snow is increasing at a rate of $\frac{23}{10}e^{-1/4}$ cm/hr

5) $C(t) \rightarrow$ cost of heating your house

a) $C'(23) = -0.21$

At $T = 23°F$, the cost of heating your house is decreasing at a rate of \$0.21/day

b) $C(23) = 2.94 \quad C'(23) = -0.21$

$L(T) = 2.94 - 0.21(T-23)$

$C(20) \approx L(20) = 2.94 - 0.21(-3)$

$= 3.57$